THE KINETIC DEUTERIUM ISOTOPE EFFECT OF PROTON-TRANSFER REACTIONS IN SOLUTION. COMPARISON OF THE MARCUS AND THE QUANTUM-STATISTICAL MECHANICAL MODELS

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The free energy dependence of the kinetic deuterium isotope effect of proton-transfer reactions in solution is usually analysed in terms of the Marcus model. By means of modern computer techniques, the more general quantumstatistical mechanical model is also amenable to routine application. The differences in the basic assumptions of the two approaches concerning the kinetic isotope effect are discussed. Reaction series for which both the Brønsted relation and the free-energy dependence of the kinetic isotope effect are available are analysed by means of the two models using one set of parameters for both free-energy relations.

INTRODUCTION

The free-energy dependence of the kinetic deuterium isotope effect (K1E) of proton-transfer reactions in solution is usually analysed in terms of the Marcus model.¹⁻⁶ By means of modern computer techniques, the quantum-statistical mechanical model is also amenable to routine application.^{7,8} Although both treatments start with different assumptions on the origin of the KIE, they yield qualitatively analogous results on the shape of the free-energy relation of the KIE. The objective of this paper is to discuss the differences between these two models with respect to the KIE and to compare the results quantitatively. Further, the theoretical results are related to experimental data for which both the Brønsted relation and the free-energy dependence of the KIE are known.

MARCUS MODEL

Within the Marcus model the rate constant $k_{\rm L}$ of a bimolecular proton-transfer reaction is given by

$$k_{\rm L} = B \exp(-\Delta G_{\rm L}^{\pm}/k_{\rm B}T) \qquad ({\rm L} = {\rm H}, {\rm D}) \qquad (1)$$

$$\Delta G_{\rm L}^{\pm} = w_{\rm f} + (\lambda_{\rm L} + \Delta G_{\rm 0}^{\prime})^2 / 4\lambda_{\rm L}$$
⁽²⁾

$$\Delta G_0' = w_{\rm p} - w_{\rm r} + \Delta G_0 \tag{3}$$

* Most of this work was done when the author was at the Department of Chemistry of the Friedrich Schiller University, Jena, GDR.

0894-3230/90/010062-07\$05.00 © 1989 by John Wiley & Sons, Ltd. where *B* is a pre-exponential factor, $\Delta G_{\rm L}^{\pm}$ the free energy of activation, $k_{\rm B}$ the Boltzmann constant, *T* the absolute temperature, ΔG_0 the free energy of reaction, $\lambda_{\rm L}$ can be viewed as a reorganization energy ($\lambda_{\rm L}/4 =$ intrinsic barrier) and $w_{\rm p}$ and $w_{\rm r}$ are the work terms.⁹ In the case of proton transfer, equation (2) is valid only within the region $-\lambda_{\rm L} < \Delta G'_0 < \lambda_{\rm L}$; outside this region equations (4) and (5) hold.

$$\Delta G_{\rm L}^{\pm} = w_{\rm r} \qquad (\Delta G_0' < -\lambda_{\rm L}) \qquad (4)$$

$$\Delta G_{\rm L}^{\pm} = \Delta G_0 + w_{\rm p} \qquad (\Delta G_0' > \lambda_{\rm L}) \tag{5}$$

According to Marcus, the reorganization energy consists of an intramolecular part, λ_i , and a medium part, λ_o , where for the moment the index indicating the isotope is omitted.¹⁰

$$\lambda = \lambda_{\rm i} + \lambda_{\rm o} \tag{6}$$

The quantity λ_i is given by

$$\lambda_{\rm i} = (1/2) \sum_{k} f_k (\Delta d_k)^2 \tag{7}$$

$$f_k = 2f_k^{\mathrm{A}} f_k^{\mathrm{B}} / (f_k^{\mathrm{A}} + f_k^{\mathrm{B}})$$
(8)

where f_k^A and f_k^B represent the force constants in the initial (A) and final state (B) of the reaction and Δd_k is the change in bond length. The reorganization energy originating from the medium can be calculated accord-

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$$\lambda_{\rm o} = (\Delta e)^2 [(1/\varepsilon_{\rm op}) - (1/\varepsilon_{\rm s})] [(1/2a_1) + (1/2a_2) - (1/r)]$$
(9)

Equation (9) refers to spherical ions with radii a_1 and a_2 and r is the distance between the centres of the reactants. The change in charge is represented by Δe , and ε_{op} and ε_s are the optical and static dielectric constants of the solvent, respectively. If one assumes that λ_L is the only isotope-dependent quantity in equations (1)-(3), the KIE is given by

$$k_{\rm H}/k_{\rm D} = \exp(1/4k_{\rm B}T)(\lambda_{\rm D} - \lambda_{\rm H})[1 - (\Delta G_0')^2/\lambda_{\rm H}\lambda_{\rm D}]$$
(10)

The free-energy dependence of the KIE is thus represented by a bell-shaped curve with its maximum at $\Delta G_0' = 0$. The maximum isotope effect at a given temperature is determined by the difference $\lambda_D - \lambda_H$. According to equation (10), the relationship between this difference and $(k_H/k_D)_{max}$ is given by

$$(k_{\rm H}/k_{\rm D})_{\rm max} = \exp\{(1/k_{\rm B}T)[(\lambda_{\rm D} - \lambda_{\rm H})/4]\}$$
 (11)

which implies that λ_D is generally larger than λ_H . The parameters of the Marcus model can be obtained from the experimental data by performing a least-squares fit of the Brønsted relation to a parabola:

$$\log k_{\rm H} = a + b(\Delta pK) + c(\Delta pK)^2$$
(12)

The relationships between the coefficients a, b and c and the model parameters are given by equations (13)–(15).

$$a = \log B - [1/(2.3k_{\rm B}T)] \times [w_{\rm r} + (\lambda_{\rm H}/4) + (w_{\rm p} - w_{\rm r})/2 + (w_{\rm p} - w_{\rm r})^2/4\lambda_{\rm H}]$$
(13)

$$b = -[(1/2) + (w_{\rm p} - w_{\rm r})/2\lambda_{\rm H}]$$
(14)

$$c = -2 \cdot 3k_{\rm B}T/4\lambda_{\rm H} \tag{15}$$

The theory described so far can be viewed as classical. Especially in the field of electron-transfer reactions, several attempts have been made to take into account quantum-mechanical aspects (for a review, see Ref. 11). All these approaches are guided by the quantum-mechanical theory and are therefore often called semiclassical treatments, although the term 'semi-classical' may have very different meanings. For proton-transfer reactions, modifications of the classical Marcus model have also been proposed. Hupe and Pohl¹ suggested consideration of zero-point energy changes on deuteration within the reactants and the products ($\Delta \epsilon_R, \Delta \epsilon_P$). Their final equation is

$$k_{\rm H}/k_{\rm D} = \exp\{(1/4k_{\rm B}T)[\lambda_{\rm D}(1 + (\Delta G_0^{\rm H} + \Delta \varepsilon_{\rm R} - \Delta \varepsilon_{\rm P})/\lambda_{\rm D})^2] - [\lambda_{\rm H}(1 + (\Delta G_0^{\rm H}/\lambda_{\rm H}))^2]\}$$
(16)

where it is assumed that $w_p = w_r$ holds and ΔG_0^H represents the reaction free energy of the protontransfer reaction. The relationship between λ_D and λ_H is given by

$$\lambda_{\rm D}/4 = [4p + (16p^2 - 4b^2)^{1/2}]/8 \tag{17}$$

where

$$p = (\lambda_{\rm H}/4) + (1/2)(1-C)(\Delta \varepsilon_{\rm R} + \Delta \varepsilon_{\rm P}) \qquad (18)$$

and

 $b = \Delta \varepsilon_{\mathbf{R}} - \Delta \varepsilon_{\mathbf{P}} \tag{19}$

C is an adjustable parameter.

Kresge proposed combining the Melander-Westheimer principle¹² with the Marcus model.² According to this proposal, the KIE is given by

$$k_{\rm H}/k_{\rm D} = \exp\{(1/k_{\rm B}T)[\delta(\rm ZPE)^{\pm}_{\rm min} - \delta(\rm ZPE)_{\rm IS}][1 - (\Delta G_0/\lambda)^2]\}$$
(20)

where $\delta(\text{ZPE})_{\min}^{\pm}$ and $\delta(\text{ZPE})_{1\text{S}}$ are the zero-point energy changes on deuteration in a symmetrical transition state and in the initial state, respectively. Again, it is assumed that $w_p = w_r$ holds. Of course, both equations (16) and (20) can be easily generalized to the case $w_p \neq w_r$. Equations (20) and (10) are formally identical, but according to the Kresge approach the maximum isotope effect is given by $\delta(\text{ZPE})_{\min}^{\pm} - \delta(\text{ZPE})_{1\text{S}}$ and not by $(\lambda_D - \lambda_H)/4$. Moreover, λ is assumed to be independent of the isotope.

QUANTUM-STATISTICAL MECHANICAL MODEL

According to the quantum-statistical mechanical model, the bimolecular rate constant k_L for proton transfer in a linear complex can be written as

$$k_{\rm L} = 4\pi \int_{R_{\rm min}}^{\infty} \mathrm{d}RR^2 \exp\left[-U_{\rm i}(R)/k_{\rm B}T\right] W_{\rm i}^{\rm L}(R)$$
 (21)

where R is the intermolecular distance and U_i the intermolecular interaction potential in the initial state of the reaction. The monomolecular rate constant, W_i^L , is given by

$$W_{i}^{L}(R) = \sum_{m} p_{im}^{L} W_{im}^{L}(R)$$
(22)

$$W_{im}^{L}(R) = (\omega_{eff}/2\pi) \sum_{n} \mathscr{H}_{mn}^{L}(R) \exp\left[-\sum_{k=0}^{n-1} 2\pi\gamma_{mk}^{L}(R)\right]$$
$$\times \exp\left\{-\left[E_{S} + \Delta E + U_{f}(R)\right]$$
$$-U_{i}(R) + E_{in}^{L} - E_{im}^{L}\right]^{2}/4E_{S}k_{B}T\right\}$$
(23)

$$\mathscr{H}_{mn}^{L}(R) = \{1 - \exp[-2\pi\gamma_{mn}^{L}(R)]\} \\ /\{1 - (1/2)\exp[-2\pi\gamma_{mn}^{L}(R))]\}$$
(24)

$$\gamma_{mn}^{L}(R) = \left[V_{e}^{2} S_{mn}^{L}(R) \right] / \left[\hbar \omega_{eff}(k_{\rm B} T E_{\rm s} / \pi^{3})^{1/2} 2 \pi \right] (25)$$

The symbols have the following meaning: W_{im} = statespecific monomolecular rate constant, p_{im} = Boltzmann population, ω_{eff} = effective medium frequency, \mathscr{H}_{mn} = transmission coefficient, γ_{mn} = Landau-Zener parameter, E_s = reorganization energy, ΔE = free energy of reaction, E_{in} and E_{im} = energies of the proton stretching vibrational states in the final and initial state of the reaction, V_e = electron resonance integral and S_{mn} = Franck-Condon factor. A computer program PROTO was written, which calculates rate constants and the KIE according to equation (21) using a Lennard-Jones potential [equation (26)] for U_i and U_f :

$$U(R) = 4\varepsilon \left[\left(\sigma/R \right)^{12} - \left(\sigma/R \right)^{6} \right]$$
(26)

The description of the interaction of two large molecules in a solvent by means of the Lennard–Jones potential can only be of a very approximate nature. The Lennard–Jones parameters ε and σ should thus be viewed as effective quantities. The Franck–Condon factor S_{mn} for the harmonic proton stretching vibration is obtained by the method of Manneback.¹³ Equation (21) can be solved either by numerical integration or by replacing the integral by the value of the integrand at its maximum. The corresponding intermolecular distance is called the transfer distance, R_{mn} . By means of PROTO, the quantum-statistical mechanical model is amenable to routine application. Avoiding integration in equations (22) and (23) results in the following simple expression:

$$k_{1} = 2\omega_{\text{eff}} [R_{00}^{\text{L}}]^{2} \Delta R \mathscr{H}_{00}^{\text{L}} (R_{00}^{\text{L}}) \exp[-U_{i}(R_{00}^{\text{L}})/k_{\text{B}}T] \\ \times \exp\{-[E_{\text{S}} + \Delta E + U_{\text{f}}(R_{00}^{\text{L}}) - U_{i}(R_{00}^{\text{L}})]^{2}/4E_{s}k_{\text{B}}T\}$$
(27)

This is analogous to the Marcus equation, provided that λ is identified with E_s and the work terms are related to the intermolecular interaction potentials in the initial and final states of the reaction. However, it is important to note that the pre-exponential factor in equation (27) is, mainly via the transmission coefficient, dependent on the transfer distance, the temperature and the isotope. The Marcus model can thus be viewed as a classical approximation to the quantum-statistical mechanical approach. In other words, if only the transition between the vibrational ground states of the initial and final states of the reaction is dominant and the preexponential factor can be assumed to be constant, the quantum-statistical mechanical and the Marcus models should give similar results. This is true for the Brønsted relation in the vicinity of $\Delta G_0 = 0$, but does not hold for the free-energy relation of the KIE. Nevertheless, the quantum-statistical mechanical model also yields a bellshaped curve for this relation.

ISOTOPE DEPENDENCE OF MODEL PARAMETERS

The isotope dependence of the rate constants and hence of the KIE originates from the isotope dependence of the model parameters. Within the Marcus model it is assumed that the pre-exponential factor and the work terms are independent of the isotope and that the isotope dependence of the reaction free energy can be neglected. In the classical version, the isotope effect is thus due to λ_L . According to equation (10), $(\lambda_D - \lambda_H)$ has a numerical value of 1906 cm⁻¹ (23 kJ mol⁻¹) for 298 K and $(k_{\rm H}/k_{\rm D})_{\rm max} = 10$. On the other hand, according to equations (6)-(9), λ should be only slightly dependent on the isotope. The bond length changes and force constants in equation (7) are quantities which can be assumed to be independent of the isotope. However, one should be aware of the fact that the reorganization energy must be viewed as a free energy. In this case, summations over excited vibrational states would have to be performed and this would result in an isotope effect. Its value should be comparable to the isotope effect of the reaction free energy, which is neglected in equation (10). It therefore seems thus to be justified to neglect the small isotope dependence of λ_i also. Replacing H by D in one of the reactants should hardly affect the medium properties. On the other hand, it was shown within the quantum-statistical mechanical model that the transfer distances for H and D may be different and that the corresponding distance for H is generally larger than that for D.¹⁴ Provided that this effect is significant, $\lambda_{\rm H}$ should be larger than $\lambda_{\rm D}$. Hence, the isotope dependence of the reorganization energy required by the classical Marcus model is not in accordance with the physical nature of this quantity.

Within the approach of Hupe and Pohl,¹ equation (16), the KIE is due exclusively to the zero-point energy changes on deuteration. For $\Delta \varepsilon_{\rm R} = \Delta \varepsilon_{\rm P} = 0$ it follows $\lambda_{\rm D} = \lambda_{\rm H}$ and hence $k_{\rm H}/k_{\rm D} = 1$. Both the maximum value of the KIE and its position are affected by the values of $\Delta \varepsilon_{\rm R}$ and $\Delta \varepsilon_{\rm P}$. For $\Delta \varepsilon_{\rm R} = \Delta \varepsilon_{\rm p} \neq 0$, the maximum of the isotope effect is at $\Delta G_0^{\rm H} = 0$ and its value is

$$(k_{\rm H}/k_{\rm D})_{\rm max} = \exp\left[\left(1/k_{\rm B}T\right)(1-C)\,\Delta\varepsilon_{\rm R}\right] \qquad (28)$$

A typical value of the empirical parameter C used by Hupe and Pohl is 0.5. In this case for $(k_{\rm H}/k_{\rm D})_{\rm max} = 10$ and 298 K, $\Delta \varepsilon_{\rm R} = 953 \,{\rm cm}^{-1}$ is required. This corresponds to the difference in the wavenumbers for an H and D vibration. One should realize, however, that the $\Delta \varepsilon$ quantities are zero-point energy changes, thus corresponding to half of these wavenumbers. For C = 0, however, $\Delta \varepsilon_{\rm R}$ would probably have the correct order of magnitude. Hupe and Pohl¹ used the wavenumber differences and not the zero-point energies. It is likely that adopting the correct values for $\Delta \varepsilon$ would require C values near zero.

Within the approach of Kresge,² the KIE is also due to the zero-point energy changes, whereas λ is assumed to be independent of the isotope. The resulting expression for $(k_H/k_D)_{max}$ is given by

 $(k_{\rm H}/k_{\rm D})_{\rm max} = \exp\{[(1/k_{\rm B}T)[\delta({\rm ZPE})^{\pm}_{\rm min} - \delta({\rm ZPE})_{\rm IS}]\}$ (29)

This is very similar to equation (28). The quantity $\delta(\text{ZPE})_{\min}^{\pm}$ has an effect analogous to that of (1 - C).

Assuming $\delta(\text{ZPE})_{\min}^{*} = 0$, the maximum isotope effect is determined by $\delta(\text{ZPE})_{1S}$, which is identical with equation (28) for C = 0.

These semi-classical versions of the Marcus approach thus provide a mainly correct interpretation of the KIE as its origin is not due to λ but to different activation energies for H and D stemming from different zeropoint energies of the H and D vibrations.

Within the quantum-statistical mechanical model, the KIE arises from the isotope dependence of both the wavefunction and of the energies of the protonstretching vibration. In Ref. 14 it was shown that the assumption of equal transfer distances for H and D is not justified. Adopting equation (27), $(k_{\rm H}/k_{\rm D})_{\rm max}$ is thus given by

$$(k_{\rm H}/k_{\rm D})_{\rm max} = \{ [R_{00}^{\rm H}]^2 / [R_{00}^{\rm D}]^2 \} \exp\{ [U_i(R_{00}^{\rm D}) - U_i(R_{00}^{\rm H})]/k_{\rm B}T \} \{ \mathcal{H}_{00}^{\rm H}(R_{00}^{\rm H})/\mathcal{H}_{00}^{\rm D}(R_{00}^{\rm D}) \}$$
(30)

provided the shapes of U_i and U_f and the frequencies of the proton-stretching vibration in the initial and final states are equal. Equation (30) is a very good approximation to the more correct expression which would be obtained by applying equation (21) directly. Contrary to the approaches discussed so far, the maximum isotope effect is mainly governed by the transmission coefficient. As already noted, this quantity is dependent on the isotope, the transfer distance and the temperature. The decreasing value of the KIE with an increasing absolute value of the reaction free energy and thus the shape of the free-energy relation is governed by the different thermal excitations of H and D vibrations. Therefore, even though the KIE is also attributed to the isotope dependence of properties of the proton-stretching vibration, this is done in a completely different manner to the Marcus model.

RELATION TO EXPERIMENTAL DATA

It would be interesting to know if there were experimental data for which the semi-classical Marcus model and the quantum-statistical mechanical model yield contradictory results. This is obviously not the case for the free-energy dependence of the KIE. However, it is felt that the dependence of the KIE on a varying intermolecular interaction may be one example. There is ample experimental evidence that increasing steric hindrance between the reactants increases the KIE, provided the other parameters affecting the isotope effect are kept fixed. For a summary of the corresponding experimental data, see Ref. 14. Within the quantum-statistical mechanical model, increasing steric hindrance can be simply modelled by increasing the Lennard-Jones-parameter σ . This results in larger values of the KIE (see Fig. 4 in Ref. 15). Equation (30) provides a simple explanation for this fact. The dominant term is the ratio of transmission coefficients. These coefficients decrease with increasing transfer distance, but the decrease is more pronounced for D than for H and this in turn produces larger values of the KIE.

For the semi-classical Marcus approach, equation (28) or (29) has to be invoked. In order to reproduce the experimental facts, increasing steric hindrance has to be accompanied by increasing values of the zero-point energy change on deuteration. This is unlikely, however. The wavenumbers of the proton or deuteron stretching vibrations are usually only slightly affected by the introduction of bulky alkyl substituents in positions adjacent to the reaction centre. For example, Hupe and Pohl¹ used in their calculations the same values of $\Delta \varepsilon_{\rm P}$ for the bases pyridine and 2,6-lutidine. For these compounds, considerable experimental data on the influence of steric hindrance on the KIE have been reported.¹⁴ It is therefore felt that the semi-classical Marcus approach does not yield a satisfactory description of the dependence of the KIE on varying intermolecular interactions. This drawback could probably be removed by introducing a pre-exponential factor into the Marcus model which is dependent on the transfer distance and on the isotope.

On the other hand, it is well known that the Marcus model can satisfactorily reproduce the free-energy dependence of the KIE. It remains to show, however, that the quantum-statistical mechanical model also provides a reasonable description of these experimental data. This has already been demonstrated in a few papers.^{7,16} The calculation procedures used previously are basically similar to the algorithm of PROTO, although the intermolecular interaction potential was usually not explicitly taken into account. To arrive at a more consistent picture of free-energy relationships, it was tested if both the Brønsted relation and the freeenergy relation of the KIE can be described by one set of parameters. To this end the Marcus parameters $\lambda_{\rm H}$ and $w_p - w_r$ were determined by a least-squares fit of the Brønsted plot to a parabola [equations (12)-(15)]. These parameters were then used to calculate the freeenergy relationships of the KIE, which were in turn compared with the corresponding experimental data. Within the quantum-statistical mechanical model the calculations were performed by numerical intergration and assuming $\lambda_{\rm H} = E_{\rm s}$ and in a first step $U_{\rm f} = U_{\rm i}$. The curve obtained in this manner was then shifted horizontally by $w_p - w_r = U_f - U_i$, where $U_f - U_i$ was regarded as a parameter. The maximum isotope effect was used to determine the difference $\lambda_D - \lambda_H$ in the Marcus model and the Lennard–Jones parameters ε and σ in the quantum-statistical mechanical model.

In Table 1, the values of the Marcus parameters for the series studied are shown. They are only slightly different from the data given in Ref. 17. In Figures 1–3 the theoretical free-energy relationships calculated by means of the Marcus and the quantum-statistical mechanical models are compared with the corresponding experimental data. It turns out that according to the

Reaction	Coefficients	$\lambda_{\rm H}/{\rm cm}^{-1}$	$w_{\rm p} - w_{\rm r}/{\rm cm}^{-1}$	Ref.
Ethyl nitroacetate-bases	$a = 1 \cdot 8711$ $b = -0 \cdot 59619$	4380	843	20, 21
Phenylnitromethane-bases	c = -0.027306 a = -1.8655 b = -0.54708	8261	778	22
Diazoacetate-acids	c = -0.014476 a = 1.465 b = -0.50893 c = -0.069289	1726	30	6
Aromatic compounds-H ₃ O ⁺	a = -1.0455 b = -0.51727 c = -0.007772	15389	531	23
4-(4-Nitrophenoxy)butan-2-one-bases	a = 0.9219 b = 0.0 c = -0.042968	2782	- 2782	1
	$a = 2 \cdot 2378$ b = -0.42049 c = -0.014953	7998	- 1272	1 ^b

Table 1. Marcus parameters obtained by a least-squares fit of Brønsted relations to a parabola^a

^a The experimental data were statistically corrected.

^b Data for the deprotonation of ethyl nitroacetate with hydroxide and phenolate²⁰ were taken into account.



Figure 1. Comparison of experimental and theoretical freeenergy relationships of the KIE. (a) Ethyl nitroacetate-bases. From (Ref. 20; solid line, quantum-statistical mechanical model; dashed line, Marcus model; the prime in $\Delta pK'$ indicates statistically corrected values; $\lambda_D = 6129$ cm⁻¹;

usual criteria, both models can reproduce the data. The agreement between experiment and theory is, however, generally not very good. In Fig. 1a for the deprotonation of ethyl nitroacetate the quantum-statistical mechanical model gives the better result, whereas in Fig. 1b for the deprotonation of phenylnitromethane the Marcus result is the better one. For very low values of E_s the quantum-statistical mechanical model yields oscillations for the free-energy relation (Fig. 2b). This reflects one of the basic features of the theoretical approach, which in an analogous version is also appropriate for the description of optical transitions. The oscillations are thus conceptually related to the vibrational structure of electronic spectra. However, for chemical reactions in solution and at room temperature, many mechanisms are probably operating which prevent the experimental observation of this phenomenon. A further result is that the quantum-statistical mechanical model gives generally less curved relationships; see also Ref. 18. It is important to note that the curvature in this model is also affected by quantities other than E_s . For example, a higher value of the electron resonance integral V_e results in a less curved relationship. The same phenomenon is observed if the anharmonicity of the proton-stretching vibration is taken into account. Nevertheless, the main effect is due to variations, in E_s . Let us now focus on the data shown

 $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 305 \text{ pm}$, $V_{\varepsilon} = 10 \text{ cm}^{-1}$, T = 300 K, $\omega_{\text{eff}} = 5 \text{ cm}^{-1}$, $\hat{v}_{\text{H}} = 3000 \text{ cm}^{-1}$). (b) Phenylnitromethanebases. From Ref. 22; $\lambda_{\text{D}} = 10170 \text{ cm}^{-1}$; $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 308 \cdot 5 \text{ pm}$; here and in the following figures other data are the same as in Fig. 1a



Figure 2. Comparison of experimental and theoretical freeenergy relationships of the KIE. (a) Aromatic compounds-H₃O⁺. From Ref. 23; $\lambda_D = 17211 \text{ cm}^{-1}$; $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 307 \text{ pm}$. (b) Diazoacetate-acids. From Ref. 6; $\lambda_D = 4438 \text{ cm}^{-1}$; $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 302 \text{ pm}$

in Fig. 3, referring to the deprotonation of 4-(4-nitrophenoxy)butan-2-one.¹ In the same way as described above, Hupe and Pohl determined for this reaction the Marcus parameters from the Brønsted relation, calculated the free-energy relationship of the KIE, compared it with the experimental data, observed a discrepancy and thus proposed the modified Marcus model mentioned earlier. On closer examination, it turned out that the authors had considered two additional reactions for the free-energy relationship of the KIE, namely deprotonation of ethyl nitroacetate by hydroxide and phenolate, without taking them into account in the Brønsted relation. The parameter estimation was thus repeated both with and without these reactions. The corresponding parameters and free-energy relationships are shown in Table 1 and Fig. 3. In both cases, the experimental data are reproduced. Hupe and Pohl, 1 however, compared the experimental data shown in Fig. 3b with the Marcus curve in Fig. 3a. Hence, the argument presented by Hupe and Pohl in favour of a



Figure 3. Comparison of experimental and theoretical freeenergy relationships of the KIE for the deprotonation of 4-(4-nitrophenoxy)butan-2-one.¹ (a) $\lambda_D = 4309 \text{ cm}^{-1}$; $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 300 \text{ pm}$. (b) Taking into account deprotonation of ethyl nitroacetate by hydroxide and phenolate (x); $\lambda_D = 9506 \text{ cm}^{-1}$; $\varepsilon = 30 \text{ cm}^{-1}$, $\sigma = 300 \text{ pm}$

modification of the Marcus model is the result of inconsistent reasoning.

There is, however, a reaction series, namely the protonation of a monoprotonated cryptand by acetic and benzoic acids, for which the Brønsted and the freeenergy relationships of the KIE cannot be described by one set of parameters.¹⁹ To date no explanation of this fact can be be given.

CONCLUSIONS

Within the semi-classical Marcus model, the kinetic isotope effect is due to different zero-point energies of the H and D stretching vibrations. Although the quantum-statistical mechanical model also attributes the isotope effect to the isotope dependence of the proton-stretching vibration, this is done in a completely different manner. The shape of the free-energy relationship of the KIE is governed by the different thermal excitations of the H and D vibrations, owing to their different wavenumbers. On the other hand, the maximum isotope effect is mainly determined by the ratio of the transmission coefficients. Although both models yield qualitatively analogous results for the shape of the free-energy relationship of the KIE, the results for the influence of varying intermolecular interactions on the KIE are different.

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